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A Reevaluation of Price Elasticities for Irrigation Water

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The effectiveness of pricing systems in the allocation of irrigation water is linked with the price elasticity of demand of farmers for water. Using microeconomic theory, it is shown that omission of the elasticity of demand for the crop produced leads to an inelastic bias in the demand for irrigated water. Linear programing approaches omit the product elasticity of demand and are consequently biased, whereas quadratic programing approaches to estimating derived demands for irrigation water include product demand functions. The difference between the resulting estimates are empirically demonstrated for regional derived demand functions estimated from a model of California's agricultural industry.

Agricultural water supplies in the west are entering an era of physical scarcity and higher relative cost. Higher energy costs for extraction and transportation coupled with reduction in the public subsidization of new agricultural water projects are changing the cost of water in relation to other agricultural inputs. In addition, the development of new sources of supply is inhibited by environmental and political constraints. These forces may result in increasing social gains to be obtained from reallocation of the current agricultural water supplies to their most efficient use within agriculture. The major obstacle to reallocation is that current water institutions in the west tend to be inflexible and are designed to avoid use conflicts rather than promote economically efficient use. Economists have long argued that alternative institutions of limited or full market pricing would enable reallocation of water supplies to more efficient use without loss of wealth to the original beneficiaries. In at least some cases it is likely that the benefits of reallocation will exceed the social costs of institutional changes.

A contrary and well-established view that *Kelso* [1967] has characterized as 'the water-is-different syndrome' maintains that the characteristics of water as a productive resource lead to extensive market failure and that even if these problems could be internalized, pricing systems will be largely ineffective as allocation mechanisms for irrigation water. The substance of the argument is that the derived demand for irrigation water is price inelastic (the price elasticity of demand is defined as the percentage change in quantity demanded resulting from a 1% change in price, that is, $\eta = -(\Delta q/\Delta P)\ (P/q)$, where P is price and q is quantity), and thus changes in prices will redistribute income to or from farmers but not alter water usage in agriculture. However, a significant problem exists in empirically testing this hypothesis. The absence of observations over a wide range of prices has necessitated the use of programing approaches to estimate the elasticities of the derived demand for water. Linear programing studies by *Moore and Hedges* [1963], *Heady et al.* [1973], and *Hedges* [1977] have tended to support the contentions that the demand for irrigation water is inelastic. However, *Shumway* [1973], also using linear programing, in a study of a large subregion of the California Central Valley found that the demand for irrigation water was elastic in the price range of $9–16 per acre-foot in 1965 relative prices. *Shumway* [1973], in reviewing the linear programing methodology used in his and previous studies, concluded that

> among the five variables affecting the demand for resources, the most heroic assumption in this study is that product demand for the entire region is perfectly inelastic.

A report to the *California Department of Water Resources* [1977] on the use of pricing to achieve water conservation concluded that

> the indication from the previous research into the impacts of water prices on agricultural water demand generally shows a relatively inelastic demand relationship though admittedly the evidence is not conclusive.

In the following section this paper shows that published estimates of price elasticity of demand for agricultural water (in the west), which have exclusively used linear programing, theoretically underestimate the elasticity of the derived demand. A quadratic programing approach is advanced as a method of correcting this bias. In the third section a quadratic programing model of California's Central Valley, the state's major agricultural region, is used to estimate empirically the arc elasticities of the derived demand for irrigation water, under both the linear and quadratic specifications. The empirical results support the theoretical findings, suggesting that the demand for water is indeed elastic over relative price intervals that are rapidly being approached in California under rising relative energy prices. The paper concludes with some brief comments on the policy significance of the elasticity findings.

Regional Derived Demands From Programing Models

Regional programing models are, by their very nature, empirical abstractions. This brief excursion into economic theory shows that the assumptions inherent in linear programs bias

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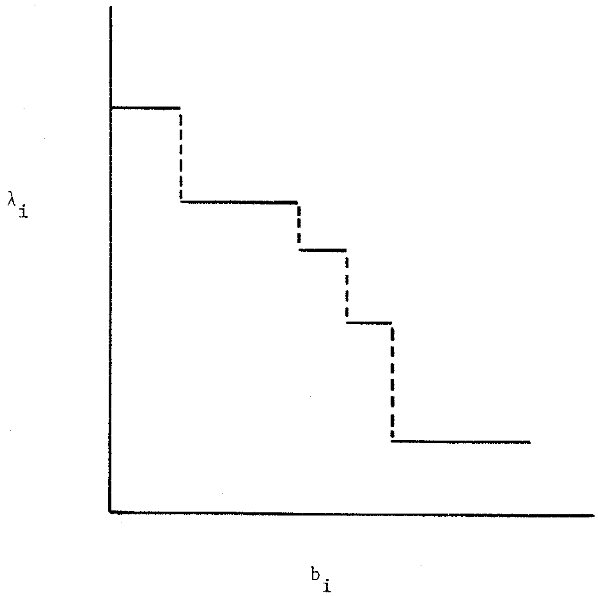

Fig. 1. Schedule of value of marginal products.

the resulting arc elasticities of derived factor demands towards inelasticity.

THE INDIVIDUAL FIRM

The derived demand for firm inputs are based on the firm profit function:

$$\pi = P_y\phi(x_1 \cdots x_n) - \sum_{i=1}^{n} P_i x_i - c \tag{1}$$

where P_y is the output price, $\phi(\ )$ the production function, x_i the inputs, and c the fixed cost. Assuming that an interior maximum exists and that the firm operates where there are diminishing marginal returns to inputs, the second-order conditions hold, and the first-order conditions are

$$P_y \frac{\partial \phi}{\partial x_i} = P_i \qquad i = 1, 2, \cdots, n \tag{2}$$

That is, the value of the marginal product (VMP) of each input is equated to input price, or marginal input cost. The system of (2) can then be solved in terms of the input quantities to yield the derived demand functions:

$$x_i = h_i(P_y, P_1, P_2, \cdots, P_n) \qquad i = 1 \cdots n \tag{3}$$

Clearly, the characteristics of the derived demand for an input depend on the price of the output as well as the relative prices of other inputs. For the case of the single firm the linear programing methodology of using the schedule of value of marginal product at different input levels to obtain the derived demand is satisfactory. However, for regional input derived demands the output price cannot be assumed constant and the quadratic programing approach which incorporates the output demand functions will yield a result that is based on a schedule of marginal revenue products at different input levels which incorporate the effects of changes in output price.

THE REGIONAL DERIVED DEMAND

Characteristics of the regional demand are derived using the Marshall-Hicks approach (detailed derivation of these relations can be found in the works by *Allen* [1968] or *Ferguson* [1971]). We now posit an aggregate production function, homogenous of degree one and simplified to two inputs, water W and land L. The regional profit function is

$$\pi_R = P_y f(L, W) - P_L L - P_w W - c \tag{4}$$

The producers in the region face a market demand for their product:

$$q_y = g(P_y) \qquad g'(P_y) < 0 \tag{5}$$

Thus in addition to the two first-order conditions analogous to (2) there is a market clearing equation:

$$q_y = f(L, W) = g(P_y) \tag{6}$$

The two first-order conditions and (6) are differentiated partially with respect to P_w, and the expressions for the elasticity of substitution σ and the elasticity of output demand η are substituted into the resulting expressions:

$$\sigma = \frac{(\partial f/\partial L)(\partial f/\partial W)}{q_y(\partial f/\partial L\partial W)} \qquad \eta = \frac{dg(\ )}{dP_y}\frac{P_y}{q_y} \tag{7}$$

The resulting set of equations is solved using Cramer's rule, and the elasticity of the derived demand for water is shown to be

$$\theta_w = \left(1 - \frac{P_w W}{P_y q_y}\right)\sigma + \frac{P_w W}{P_y q_y}\eta \tag{8}$$

The weighting expression $P_w W/P_y q_y$ in (8) is the proportion of regional revenues needed to pay water costs. It follows from (8) that the higher the cost of water becomes in relation to output prices, the more important is the contribution of the elasticity of demand of the output to the derived demand elasticity for the input. In addition, the linear programing formulation of the problem omits the second term in the right-hand side of (8). Constant price horizontal demand curves are conventionally referred to as infinitely elastic, which can be seen from the definition of η to be true in the limit ($\Delta P \rightarrow 0$). However, in the empirical case using linear programing, $\Delta P = 0$, and η is therefore undefined. Thus the linear programing approach underestimates the elasticity of derived demand for the input.

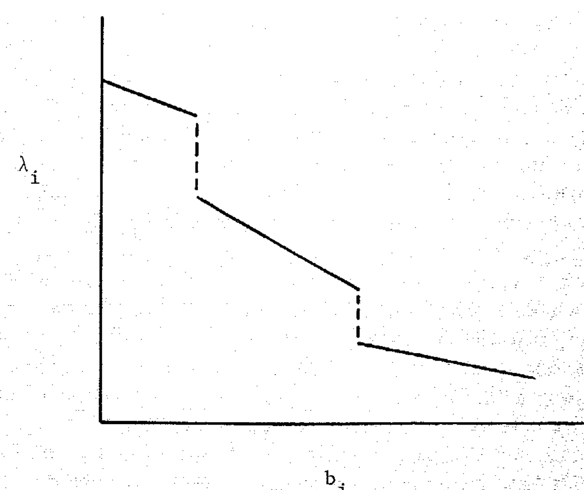

Fig. 2. Schedule of marginal revenue products.

THE PROGRAMING APPROACH

Unfortunately, these neoclassical results, which assume the existence of an interior solution for every production function, are not directly applicable to either linear or quadratic programing. While expressions (1)–(8) characterize derived demands, the linear fixed proportion production functions that define the convex set of production possibilities in linear (LP) and quadratic programing (QP) have, individually, no interior solution. Consequently, the elasticity of substitution as defined in (7) does not exist for any activity.

Despite the fixed proportion restrictions on individual activities, the solutions from aggregate QP and LP production models do alter the aggregate output mix in reaction to changes in scarce factor availability. The substitutions are effected by changes in the optimum levels of activities in the basis (acres of crops and tons produced) and by changes in the basis of optimal activities. That is, production activities are shifted in and out of the optimal basic solution in response to changes in the 'right-hand side' resource constraints. As the optimal solution changes, so will the dual values associated with the constraining resources. By plotting the schedule of imputed dual prices of a resource input against a range of available quantities of the resource a derived demand for the resource can be determined for scarcity levels and values that are not yet reflected in historical data. The resulting derived demand is a schedule and not a function, since it is plotted through a series of points and discontinuities at each change of basis. The change of slope of this schedule yields an estimate of the empirical elasticity of derived demand for the input and depends on both the slope of the schedule for changes within a given basis and the discrete discontinuous jumps at a change of basis. Derivation of the schedules under the LP and QP specifications demonstrates the theoretical reason why the input derived demand arc elasticities are more elastic when derived from quadratic programs under identical production functions. The linear programing problem can be concisely specified as

$$\max c'x \tag{9}$$

subject to

$$Ax \le b \qquad x \ge 0$$

where c is a $n \times 1$ vector of output prices and x is an $n \times 1$ vector of activity levels that can be set at any nonnegative value. The $m \times n$ matrix A can be thought of as n $m \times 1$ vectors of coefficients from linear production functions, relating the amount of each of the m input factors needed to produce a unit of output. The $m \times 1$ vector b represents the maximum or minimum resource quantities available for regional production. Utilizing the notation from *Hadley* [1962], the optimal basic solution to (9) can be expressed as

$$x_* = B_L^{-1}b \tag{10}$$

where the matrix B_L is an $m \times m$ matrix of those columns of A that correspond to activities in the basic linear programing solution. The $m \times 1$ dual vector λ is interpreted as the value of marginal products of the constraining resources and is defined as

$$\lambda' = c_*'B_L' \tag{11}$$

where c_* is defined as an $m \times 1$ partition of the output price vector c associated with the basis activities x_*. Within any basis the effect of resource quantity changes on the value of marginal product (VMP) is zero:

$$d\lambda/db = 0 \tag{12}$$

The schedule of VMP's derived from a linear program in Figure 1 is composed of a series of horizontal functions with discontinuous vertical shifts marking the change of basis.

Analogously, the quadratic programing problem is specified as

$$\max\ [d - \tfrac{1}{2}Dx]'x \tag{13}$$

subject to

$$Ax \le b \qquad x \ge 0$$

where the objective function contains a set of n linear demand functions for the outputs x_i, $i = 1 \cdots n$. The elements of d, d_i are the demand intercepts, while the $n \times n$ matrix D is composed of own price slope coefficients on the diagonal and cross effects, if any, on the off diagonal elements. The optimal basic solution is derived as

$$x_* = B_Q^{-1}b \tag{14}$$

where B_Q is the subset of A which is the basis matrix for the quadratic solution.

By formulating (13) as a Lagrangian and applying Kuhn-Tucker conditions the constraint for the dual quadratic problem is derived as

$$\lambda'A + Dx \ge d \tag{15}$$

Thus using the profit maximizing assumption, the quadratic dual values which are an $m \times 1$ vector of marginal revenue products can be expressed as

$$\lambda' = (d_* - D_*x_*)B_Q^{-1} \tag{16}$$

where the asterisk subscript refers to the basis dimension and B_Q is the basis matrix for the quadratic problem. Substituting (14) into (16) and taking the derivative with respect to b, we obtain

$$\frac{d\lambda}{db} = -B_Q'^{-1}D_*B_Q^{-1} \tag{17}$$

Expression (17) shows that within any given basis, or set of crops in the agricultural producer's case, the producer's valuation of water resources will indeed change in response to changes in the quantities available and that they will do so in the expected direction. The resulting schedule of marginal revenue products illustrated in Figure 2 still has vertical discontinuities where the basis changes, but the resource use b_i is responsive to changes in resource value λ_i within any given basis. Direct comparison of the expressions for λ in (11) and

TABLE 1. Quantities of Water Demanded by Central Valley Region

Water Price per Acre-Foot	Water Quantity, 1000 ac ft	
	LP Specification	QP Specification
\$25	5099.86	6952.37
\$30	4005.86	4607.42
\$35	3670.05	4118.73
\$40	3578.96	3969.04
\$45	3493.03	3670.05

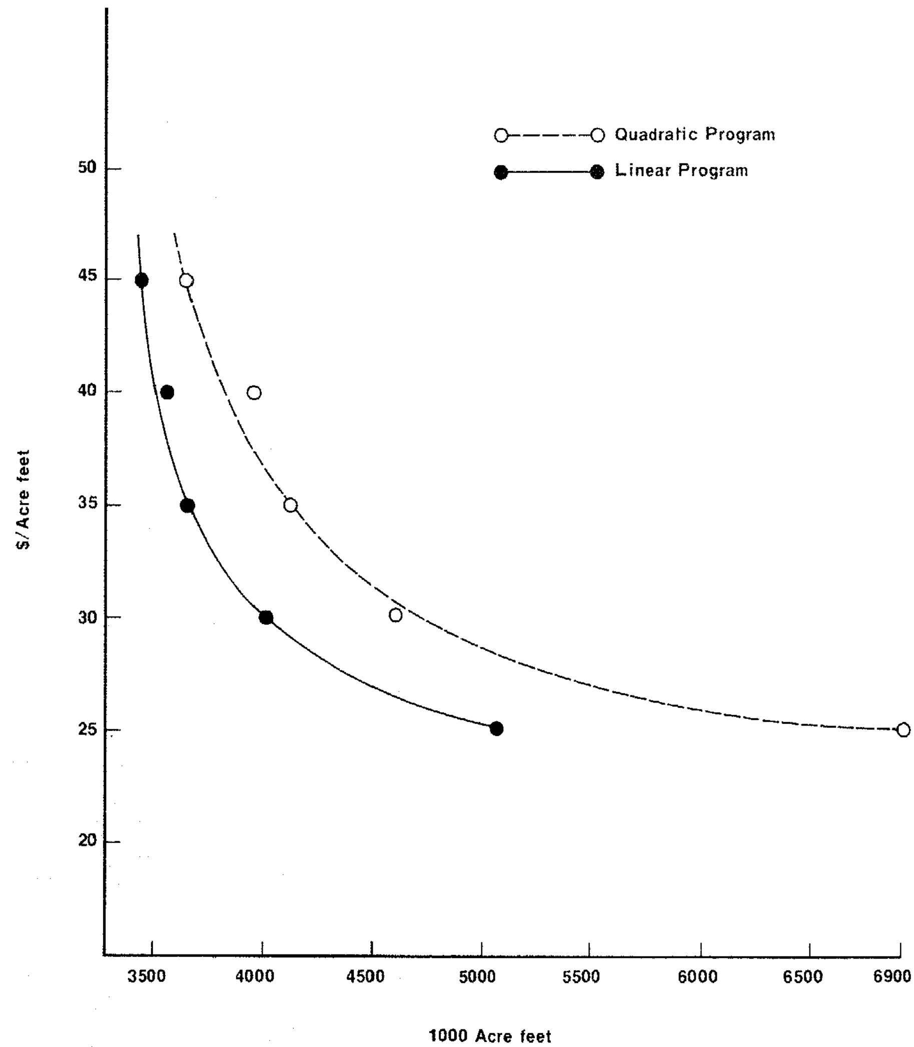

Fig. 3. Derived demands for irrigation water.

(16) is not possible despite identical initial production specification because the basis set of activities that optimizes the objective function cannot be assumed to be the same. Furthermore, the changes of basis cannot be assumed to be identical.

From a regional water policy viewpoint the noncomparable LP and QP basis sets are less of a handicap. Changes of basis will occur when the optimal set of activities change, that is, when certain crops are abandoned by all growers in the district or new crops are introduced. In the short and medium run one would expect the producer reaction to relative water price changes to take the form of shifts in the relative acreages in various regional crops, which does not necessarily involve a change of basis. (The need to change basis when input factor ratios are changed depends on how the activities are specified.) Thus for short-run adjustments in relative crop acres the quadratic programing specification, which more closely approximates the theoretical derived demand (3), will yield more elastic arc elasticities for factor inputs than the linear programing specification.

TABLE 2. Arc Elasticities of Demand for Irrigation Water

Arc Price per Acre-Foot (1976 Dollars)	LP Specification	QP Specification
$25–35	0.9717	1.502
$35–45	0.1982	0.4622

In the next section we test this hypothesis by generating the derived demand schedules for irrigation water in California's Central Valley from a programing model that is run under both linear and quadratic programing specifications.

AN EMPIRICAL APPLICATION TO ANNUAL CROPS IN CALIFORNIA

The statewide quadratic programing model of field crop production is a modified version of the model developed by *Adams et al.* [1977]. The set of energy rationing constraints necessary to the Adams study was relaxed to allow interregional transfers, and groundwater and surface water are assumed perfect substitutes up to specified annual quantity constraints. Variable costs by region for groundwater and surface water delivery are also incorporated.

The objective function maximizes the sum of consumer's surplus and producer's quasi-rent over the aggregate demand functions for 9 field crops and 28 seasonal vegetable crops. Perennial crops are not included in the model. For short-run optimal agricultural water allocation decisions it is assumed that the capitalized loss to the individual grower and the state associated with the stressing of tree and bush crops due to water deficiencies exceeds the annual opportunity cost of water used for field crops. Thus perennial crops are afforded full water allocations.

The statewide field crop production potential is divided into 14 regions, each with two irrigable soil types. The regions are based on homogeneity of climate, soil types, and water costs and availability and are described by *Adams* [1975]. The model considers 370 possible quarterly production activities and is constrained only by the availability of land, water, and energy in the regions and, in a few cases, processing capacity. The regional yields and production costs used in the derivation of farm income are based on University of California Agricultural Extension Service estimates [*University of California,* 1978].

The model has the acknowledged drawbacks inherent to normative models based on the assumption of optimizing behavior by all individuals and the pitfalls of aggregate data collection. With these limitations the model is a comprehensive short-run model of California agriculture which specifically accounts for shifts in production mix, surface/groundwater ratios, and for the price effects relating to many of California's volatile but valuable vegetable crops.

To make the derived demand schedules directly comparable at discrete points, the value-to-quantity relationship was derived by changing the cost of water in the objective function over an aggregate range that is relevant for future California water policy, i.e., from \$25 to \$45. It can be seen from the first-order conditions (2), that the VMP or marginal revenue product (MRP), respectively, will be equated to the aggregate water cost. (Marginal revenue product is defined as the product of marginal revenue and marginal physical product. Thus the MRP is the dual value associated with binding resource constraints in a QP.) Comparison of LP and QP formulations under identical production technology and constraints is easily achieved in the algorithm employed by setting the quadratic term coefficients to zero and adjusting the intercept for the LP results. The quadratic program algorithm used is by *Best and Ritter* [1975] and uses a linear programing subroutine to solve the first stages of the quadratic problem. Prices were set equal to those averaged in California.

Although the model covers all agricultural regions in California, the area selected for water price changes was the Central Valley, that is, both the Sacramento and San Joaquin basins. The Central Valley area grows the preponderance of California annual crop production, although the effect of water cut backs will be somewhat reduced by substitution from coastal areas.

The quantities of water demanded at various prices under alternative model specifications are shown in Table 1, and the two derived demand schedules are plotted on Figure 3.

Of direct interest is the comparison of the arc elasticities of demand for irrigation water under the alternative specifications derived from Table 1 and shown in Table 2. The downward bias in arc elasticities derived from linear programing models that is theoretically anticipated in the first section is clearly shown in the results in Table 2.

CONCLUSION

The downward bias of derived demand elasticities for irrigation water previously estimated by LP methods has been demonstrated in the case study to change the empirically estimated demand from slightly inelastic to significantly elastic in the \$25–35/ac ft price range. This is, the price range within which the aggregate relative price of irrigation water might be expected to increase over the next 10 years. It should be noted that the programing model employed can only be considered valid over this medium run period. However, the elastic demand for irrigation water over this price range has significant policy implications in the areas of allocation during droughts or shortages, future supply development, and future repayment capacity of existing systems.

Under conditions of drought, the policy of allocating shortfalls in supply by administrative fiat leads to conflicts between equity and efficiency considerations. Under elastic derived demand conditions and a controlled pricing and transfer scheme to minimize externalities, water can be sold to its 'highest and best use' and simultaneously can yield greater returns per acre-foot to the original owner of the water right. In the same vein, permanent future price increases in irrigation water, made inevitable in the Central Valley by renegotiation of power contracts, are forecast to raise the average cost of water delivered by the state water project into the \$25–35 range [*California Department of Water Resources,* 1978]. If the estimated elastic demand is correct, the cost increase will reduce the quantity demanded by existing users and thus will tend to offset the current predictions of severe supply shortfalls calculated under the assumption of inelastic 'needs' for agriculture [*California Department of Water Resources,* 1977].

Less inviting is the prospect for district repayment capacities that is implied by elastic derived demands and increasing costs of water delivery. As the cost of delivery rises and farmers reduce the quantities of water demanded, districts faced with fixed debt obligations will find their revenues falling. One alternative is to further raise the cost of water to cover the revenues; this will only be effective as the demand gets increasingly inelastic at high prices. A more attractive, though institutionally restricted alternative, is to sell the excess obligated water.

The accuracy of estimates of price elasticities of demand for irrigation water is thus extremely important in predicting producer reactions to future changes in relative water prices. While the quadratic programing methodology used in this paper could be improved on through the incorporation of perennial crops and alternative irrigation technologies, it may be viewed as an improvement over the biases inherent in previous LP methodologies.

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